A transient fluctuation theorem in closed quantum systems

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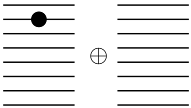
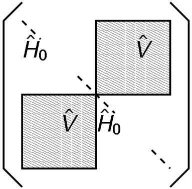
How do closed quantum systems approach equilibrium?

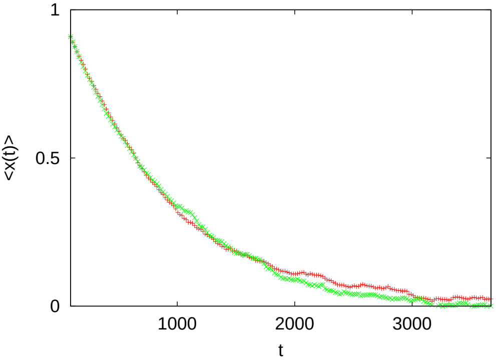
"two-site hopping model"



Hamiltonian in matrix form:



hopping amplitudes $\hat{V}$: Gaussian random numbers
local spectra $\hat{H}_0$: equidistant, N levels
observable: $\hat{x} = \hat{\Pi}_L - \hat{\Pi}_R$



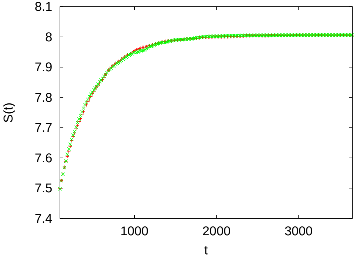
-For a certain parameter range concerning hopping strength, bandwidth, state density, etc. the "position observble" $\langle x(t) \rangle$ may relax exponentially according to Fermi's Golden Rule.
-There is "dynamical typicality", i.e., evolutions $\langle x(t) \rangle$ from pure states featuring the same $\langle x(0) \rangle$ do not look very different.

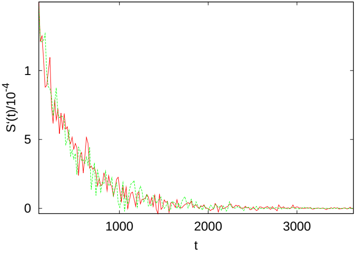
What about entropy?

von Neumann entropy of the full state never changes:
$S_{vN.-tot.}(t) = 0$, $\Rightarrow$ no 2. Law.
But, say you only observe "particle-right" or "particle-left". $\Rightarrow$ construct the maximum entropy state consistent with that information $\Rightarrow$

"reduced state": $$\hat{\rho}_{red.} = \frac{1}{2N}((1 + \langle x \rangle)\hat{\Pi}_L + (1 - \langle x \rangle)\hat{\Pi}_R$$

Now consider von Neumann entropy of reduced state $S_{vN.-red.}(t) = 0$:





What about fluctuations of mean entropy production?

The transient fluctuation theorem (TFT) (roughly) states for non-equilibrium states:

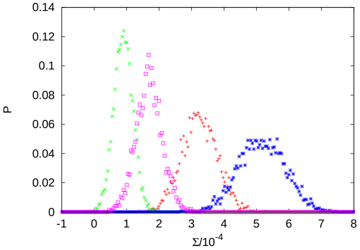
$$\frac{P_t(\Sigma(\tau))}{P_t(-\Sigma(\tau))} = e^{\tau\Sigma(\tau)}$$

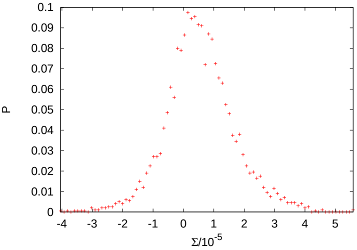
$P_1(\ldots)$: probability of $(\ldots)$ at time(range) t
$\Sigma(\tau)$: entropy production averaged over τ

Consider for example a Gaussian distribution in accord with the TFT

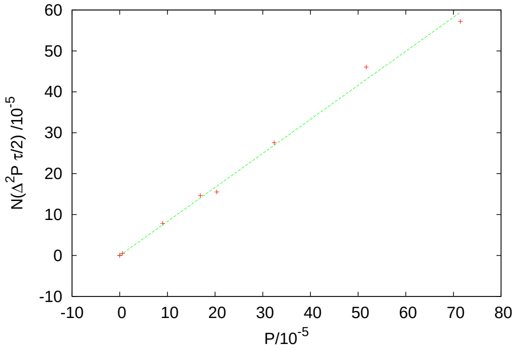
$$P_t(\Sigma(\tau)) \propto e^{\frac{\tau(\Sigma-\Sigma_0(t))^2}{4\Sigma_0(t)}}, \quad \overline{\Sigma(\tau)} = \Sigma_0(t), \quad \Delta^2\Sigma(\tau) = \frac{2\Sigma_0(t)}{\tau}$$

"Statistics" of our model system:





mean average entropy production vs. variance of average entropy productions:



Variance scales linearily with mean entropy production ⇒ in accord with TFT
What about prefactors? ⇒ looks like a factor of N is missing. ⇒

$$S(\langle x \rangle) := N\, S_{vN.-red.}(\langle x \rangle)$$

Why should one consider interpretations of the quantum dynamcis in terms of stochastic processes?

approximation to the entropy: $S(\langle x\rangle) \approx N\ln(N) - \frac{1}{2}N\langle x\rangle^2$

Assume $\langle x_i\rangle := \langle x(i\tau)\rangle$ was given by the following Ornstein-Uhlenbeck process

$$d\langle x_i\rangle := \langle x_{i+1}\rangle - \langle x_i\rangle = -R\,\langle x_i\rangle\,\tau + \sqrt{2R\tau/N}\cdot y_i$$

with y_i being Gaussian distributed, uncorrelated "stochastic increments" featuring

$$\overline{y_i} = 0 \quad \overline{y_i y_j} = \delta_{ij}.$$

$\Rightarrow$ Then it can be shown quickly that a transient fluctuation theorem directly follows.

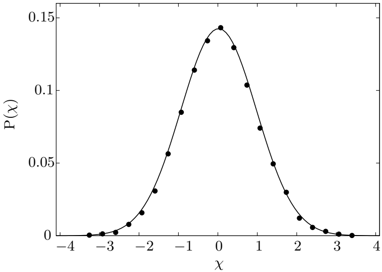
Do the $\langle x_i\rangle$ as generated by the Schroedinger equation show similarities with the Ornstein-Uhlenbeck process?

$$\langle\psi|x(t+\tau)|\psi\rangle - \langle\psi|x(t)|\psi\rangle = -R\langle\psi|x(t)|\psi\rangle\tau + \sqrt{2R\tau/N}\,\chi(\psi,t,\tau)$$

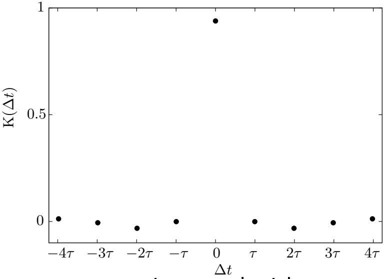
We analyze the "statistics" of the $\chi(\psi,t,\tau)$

Results on the statisitics of the increments $\chi(\psi, t, \tau)$ for a single trajectory numericaly computed from the Schroedinger equation:

relative frequency of stochastic increments χ



correlation of stochastic increments
$K(\Delta t) := \overline{\chi(\psi, t + \Delta t, \tau)\chi(\psi, t, \tau)}$



Relative frequencies and correlations of the increments are in accord with a stochastic interpretation.

Just an incidence for this model?

$\Rightarrow$ Consider any model + observable for which $\langle x(t)x(0)\rangle \propto e^{-Rt}$

Statistics of increments along a single trajectory are impossible to determine without solving the full Schroedinger equation.

But statistics of increments whith respect to a (reasonable) ensemble of pure states may be computed analytically solely on the basis of the above correlation function.

This may be done using so called Hilbertspace Averages (HA[...]):

$$HA[\chi(\psi, t, \tau)] \approx 0 \qquad HA[\chi(\psi, i\tau, \tau)\chi(\psi, j\tau, \tau)] \approx \delta_{ij}$$

Given a form of ergodicity, this idicates that the above Ornstein-Uhlenbeck description of the quantum exepectation value dynamics is generic!

If you are interested in references just ask me or write me an email.

Thank you very much for your attention!