Generalized reduced entropy and fluctuation theorems in closed quantum systems

Christian Bartsch, Jochen Gemmer

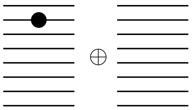
University of Osnabrück,

International Symposium on Quantum Thermodynamics

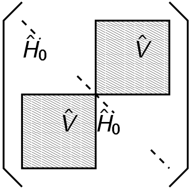
Stuttgart, Sept. 17., 2010

How do closed quantum systems approach equilibrium?

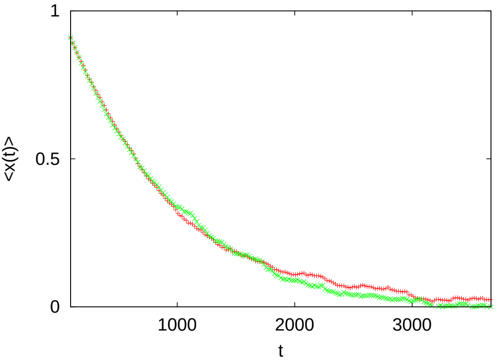
"two-site hopping model"



Hamiltonian in matrix form:



hopping amplitudes $\hat{V}$: Gaussian random numbers
local spectra $\hat{H}_0$: equidistant, N levels
observable: $\hat{x} = \hat{\Pi}_L - \hat{\Pi}_R$



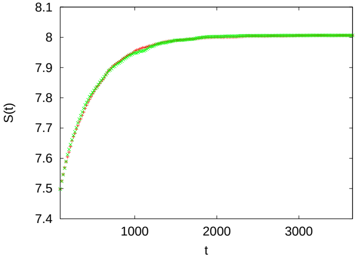
-For a certain parameter range concerning hopping strength, bandwidth, state density, etc. the "position observble" $\hat{x}$ may relax exponentially according to Fermi's Golden Rule.
-There is "dynamical typicality", i.e., evolutions $< x(t) >$ from pure states featuring the same $< x(0) >$ do not look very different.

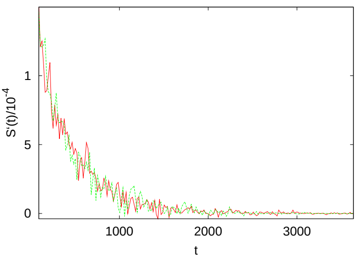
What about entropy?

Von Neumann-entropy of the full state never changes:
$S_{VN.-tot.}(t) = 0$, $\Rightarrow$ no 2. Law.
But, say you only observe "particle-right" or "particle-left". $\Rightarrow$ construct the maximum entropy state consistent with that information $\Rightarrow$

$$\hat{\rho}_{red.} = \frac{1}{2N}((1+ < x >)\hat{\Pi}_L + (1- < x >)\hat{\Pi}_R$$

Now consider Von Neumann-entropy of reduced state $S_{VN.-red.}(t) = 0$:





What about fluctuations of mean entropy production?

The fluctuation theorem (FT) (roughly) states for stationary non-equilibrium states:

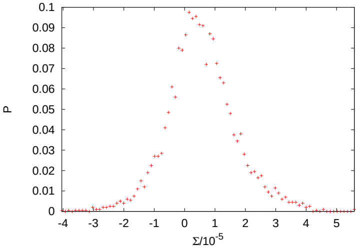
$$\frac{P(\Sigma(\tau))}{P(-\Sigma(\tau))} = e^{\tau \Sigma(\tau)}$$

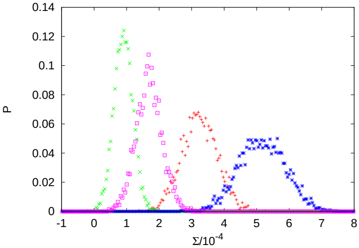
$P(..)$:relative frequency
$\Sigma(\tau)$: mean entropy production over τ

Consider for example

$$P \propto e^{\frac{1}{2}\left(\frac{(\Sigma-\Sigma_0)^2}{\frac{2\Sigma_0}{\tau}}\right)}, \quad \bar{P} = \Sigma_0, \quad \Delta^2 P = \frac{2\Sigma_0}{\tau} \quad \Rightarrow \text{in accord with the FT}$$

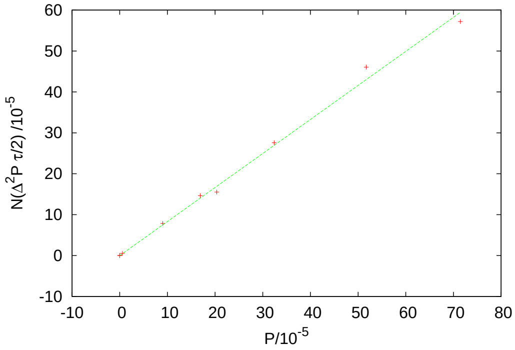
Statistics of our model system:





Do the entropy production statistics of our model fulfill the FT?

mean (mean) entropy production vs. variance of (mean) entropy production:



Variance scales linearly with mean entropy production ⇒ in accord with FT
What about prefactors? ⇒ looks like a factor of N is missing.

This may be cured by defining a "quantum-Ehrenfest-entropy", $S_{qE.}$ as

$$S_{qE.} = N\, S_{VN.-red.}(< x >)$$

Why "quantum-Ehrenfest"? $\Rightarrow$ $S_{qE}(< x >)$ turns out to be an (approximate) logarithmic meassure of the size of the compartement of Hilbertspace that corresponds to $< x >$, call that $\Gamma(< x >)$, $\Rightarrow$

$$S_{qE} \approx \ln \Gamma(< x >)$$

Whether or not this makes sense should be principally dicussed and numerically checked for other model sizes, classes, etc.

Acknowledgements

Many thanks to:

- Christian Bartsch
- the unknown people who wrote the Wikipedia entry on the fluctuation theorem
- Guenter Mahler for having been a great academic advisor in all those years
- You, the audience, for listening carefully!