Statistical relaxation behavior in closed quantum systems

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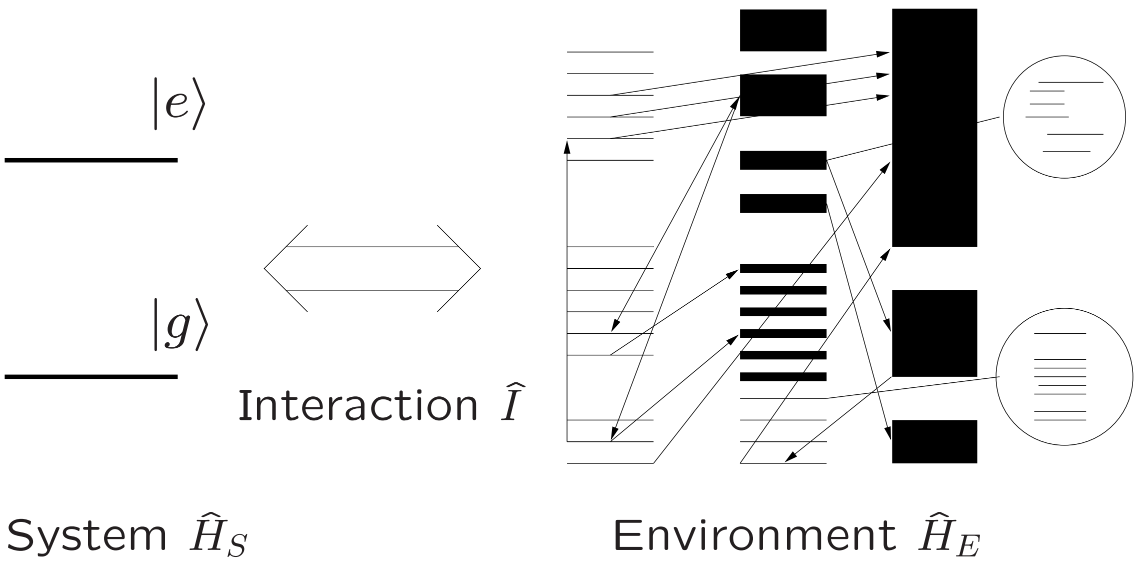
System and question

System in contact with something that has a temperature:



Hilbertspace of the environment: arbitrarily large
Concrete form of the interaction: hard to tell

Technically one would have to solve:

$$i\hbar\frac{d}{dt}|\Psi\rangle = \left(\hat{H}_S + \hat{H}_E + \hat{I}\right)|\Psi\rangle \quad \text{(very many dimensions)}$$

But one does solve:

$$\frac{d}{dt}P_e = -\gamma P_e \quad \text{(one dimension)}$$

(P_e: excitation probability)
Routinely the complexity of the problem is drastically reduced by assuming statistical rather then coherent dynamics.
Why?

Fermis Golden Rule (FGR): The dynamics of P_e for a short timestep are computed

$$P_e(\tau) - P_e(0) = -\gamma\tau P_e(0)$$

: does not allow for $|\Psi(0)\rangle$ being a superposition $\Rightarrow$ not iterable.

Weisskopf-Wigner Theorie:

$$\langle P_e|\Psi(t)\rangle = \int_0^t \langle P_e|\hat{I}(t')|\Psi(t')\rangle dt'$$

is (approximately) solved for the case of an atom (system) being coupled to the (free) electromagnetic field (environment) $\Rightarrow$ hard to generalize to other situations.

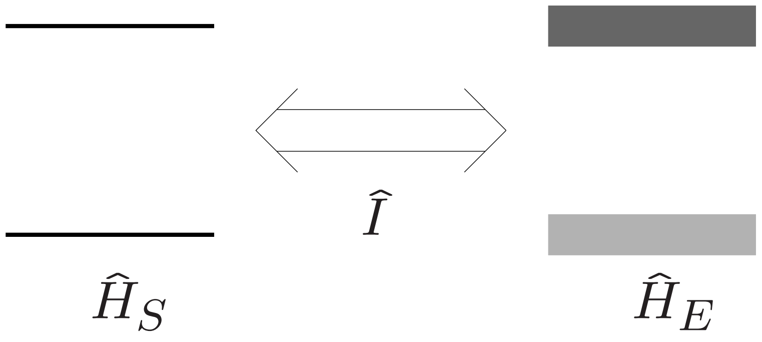
Master equations: From

$$\mathrm{Tr}_E\left\{\hat{\rho}(t)\right\} = \int_0^t \mathrm{Tr}_E\left\{\left[\hat{\rho}(t'), \hat{I}(t')\right]\right\} dt'$$

an autonomous subdynamic for $\mathrm{Tr}_E\left\{\hat{\rho}\right\}$ is computed. $\Rightarrow$ Doing so, one has to make a Markov assumption, (usually) assume a productform of the full state, i.e. $\hat{\rho} = \hat{\rho}_S \otimes \hat{\rho}_E$,
and ignore backactions of the system onto the environment.

Hilbertspace average theory

The system we investigate may be depicted as:



Just like all other approaches, we use the Dirac Picture, set up a Dyson series and truncate at second order

$$|\Psi(t+\tau)\rangle \approx \hat{U}(t,\tau)|\Psi(t)\rangle$$

For the excitation probability we get:

$$\langle \hat{P}_e(t+\tau)\rangle = \underbrace{\langle \Psi(t)|\hat{U}^{+}(t,\tau)\hat{P}_e\hat{U}(t,\tau)|\Psi(t)\rangle}_{:=\Pi(|\Psi\rangle,t,\tau)}$$

Now we compute the Hilbertspace average ($[\![...]\!]$) of the right hand side over all $|\Psi\rangle$, under the sidecondition $\langle\Psi|\hat{P}_e|\Psi\rangle = \langle\hat{P}_e(t)\rangle$.
Doing so we find

$$[\![\Pi(|\Psi\rangle,t,\tau)]\!] \approx \langle\hat{P}_e(t)\rangle - \gamma\tau\langle\hat{P}_e(t)\rangle$$

under similar preconditions needed for the applicability of FGR.

Hilbertspace average theory

Is it a good approximation to replace Π by $[\![\Pi]\!]$? $\Rightarrow$ Yes, if the "Π-landscape" is flat!

It can be shown that the "Flatness" of the Π-landscape increases with increasing number of levels of the environment, N_e $\Rightarrow$

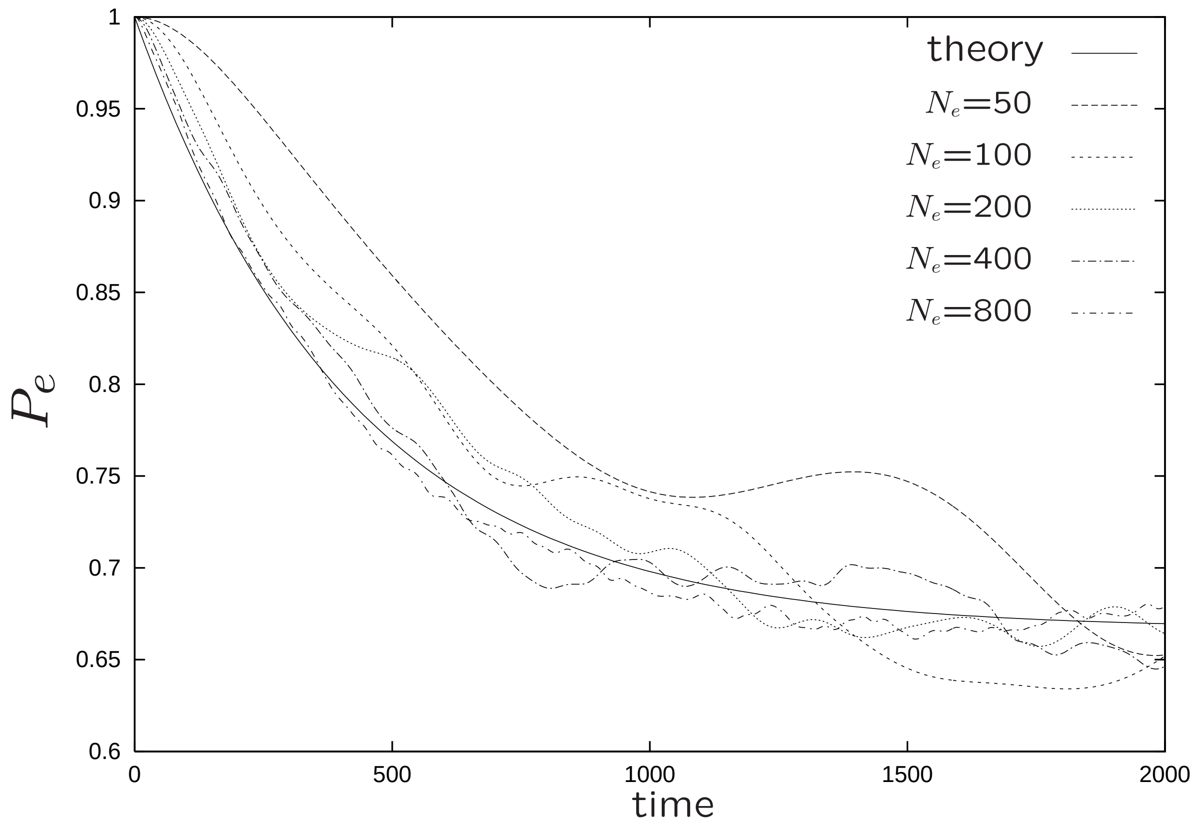
$$\langle \hat{P}_e(t+\tau)\rangle \approx \langle \hat{P}_e(t)\rangle - \gamma\tau\langle \hat{P}_e(t)\rangle$$

This is iterable, since it includes superpositions and has no explicit time dependence $\Rightarrow$

$$\frac{d}{dt}\langle P_e\rangle = -\gamma\langle P_e\rangle$$

Results:

Solving the Schrödingerequation for the above model with a interaction matrix chosen at random yields:



Results

System features:

- "intermediate systemsize"
- environment is not a set of decoupled oszillators
- system evolves into highly entangled states
- environment not in a "thermal" state
- system affects the environment

$\Rightarrow$ Nevertehless we find good statistical behavior.

More about this:

www.theo1.physik.uni-stuttgart.de $\rightarrow$ AG Mahler $\rightarrow$ Forschung

J. Gemmer, A. Otte, G. Mahler, Phys. Rev. Lett. **86**, 1927 (2001)

J. Gemmer, G. Mahler, Eur. Phys. J. B. **31** 249 (2003)

Issue of "Lecture Notes in Physics" , J. Gemmer, M. Michel, G. Mahler, Springer-Verlag, hopefully available in summer 2004