Projection operator approach to transport and relaxation in closed quantum systems

Jochen Gemmer

University of Osnabrück,

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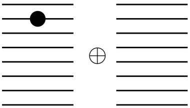
projection onto an observable: $A(t) := \mathrm{Tr}\{\hat{A}\hat{\rho}(t)\}$

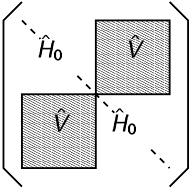
$$\mathcal{P}\hat{\rho} = \hat{1} + \mathrm{Tr}\{\hat{A}\hat{\rho}\}\hat{A} \quad \text{with} \quad \mathrm{Tr}\{\hat{A}^2\} = 1, \quad \mathrm{Tr}\{\hat{A}\} = 0 \quad \Rightarrow \mathcal{P}^2\hat{\rho} = \mathcal{P}\hat{\rho}$$

$$\text{TCL projection yields:} \quad \dot{A} = (\Gamma_2(t) + \Gamma_4(t) + \cdots)A$$

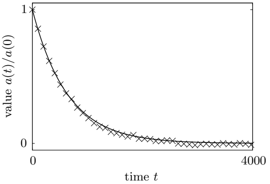
"two-site hopping model": $\tau_C \ll \tau_R$, $\hat{A}_1 = \frac{1}{\sqrt{2}}(\hat{n}_L - \hat{n}_R)$, $\hat{A}_2 = \hat{n}_L$



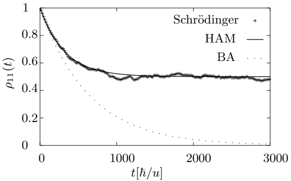
$\hat{H}_0$: local energies
$\hat{V}$: hoppings



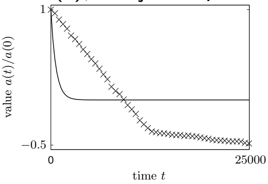
$A_1(t)$, V_{ij} random



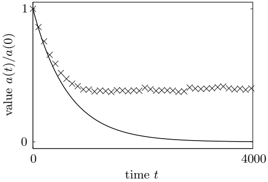
$A_2(t)$, V_{ij} random



$A_1(t)$, V_{ij} all equal



$A_1(t)$, V_{ij} sparse

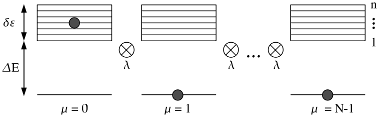


Higher order terms may be crucial, even for "weak coupling"!

projection onto density waves: $\hat{A}_q = \sum_x \cos(qx)\hat{n}(x)$

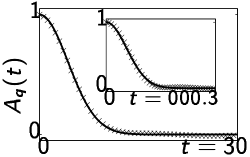
classification: $A_q(t) \propto e^{-q^2 t}$: diffusive, $A_q(t) \propto e^{-q^2 t^2}$: ballistic

many site hopping model

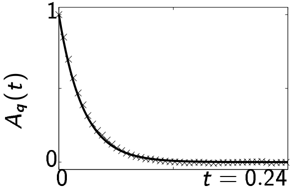


V_{ij} random, $\tau_C = 2 \cdot 10^{-3}, \tau_{rec} = 1$

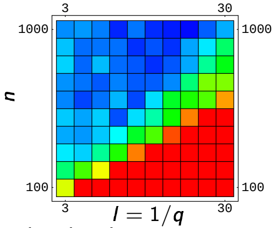
long/short wavelength



intermediate wavelength

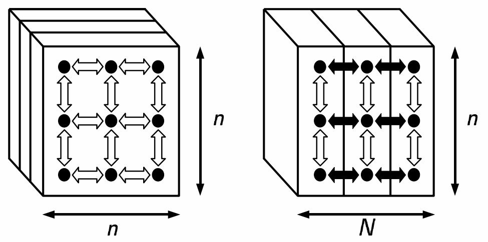


"diff.-ball. map":



Lengthscale dependent diff.-ball. transition, second order description seems to hold for all times!

3d-Anderson model

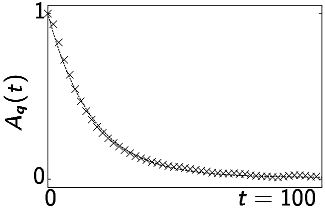


$$\hat{H}=\sum_{\mathbf{r}}\epsilon(\mathbf{r})\,\hat{a}^\dagger(\mathbf{r})\,\hat{a}(\mathbf{r})+\sum_{NN}\hat{a}^\dagger(\mathbf{r})\,\hat{a}(\mathbf{r}')+\text{h.c.}$$

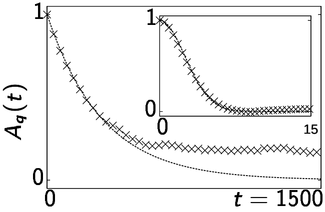
$\epsilon(\mathbf{r})$: Gaussian random numbers, σ $\Rightarrow$

$$\hat{H}=\sum_{\mu=0}^{N-1}\hat{h}_0(\mu)+\lambda\sum_{\mu=0}^{N-1}\hat{v}(\mu,\mu+1)$$

intermediate wavelength



long/short wavelength



$n=30,\ N=42,\ \sigma=1$

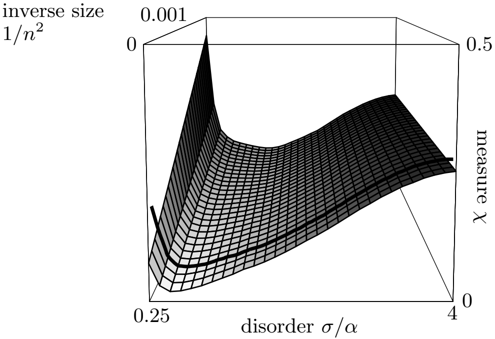
Loclization appears as fourth order effect

Size of the diffusive regime in the 3-d Anderson model

Evaluating fourth order terms is an art. We work with a feasible estimation based on the fact that there is Van Hove structure.

τ_C: transition to ballistic lengthscale, $\Gamma_{q,4}$: transition to localized lengthscale

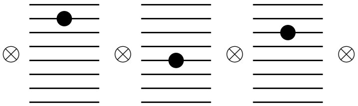
We compute $\sqrt{1/\chi}$: "range" (maximum ratio) of diffusive wavelengths in between ballistic (short) and localized (long) wavelengths.



This suggests: diffusive behavior only between l_{min}, l_{max} with $l_{max}/l_{min} \approx 7$ for infinitely sized systems and "optimum" disorder

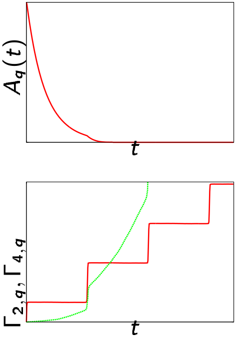
“many particle” modular quantum system:

defined on the full product space of the subunits, random NN-interactions, translational invariance, 1-d

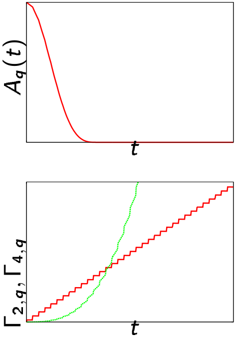


project onto local energy waves

intermediate wavelength:



long wavelength:



This indicates a lenghtscale dependent “diff.-ball.-transition” in a (strongly) interacting, 1-d, quantum chaotic system

the "take home message":

Alternative approaches to quantum transport may help to get more detailed information on transport behavior with respect to the lengthscale. Furthermore quantitative analysis of (strongly?) interacting 2-d or 3-d systems may possibly become feasible.

more information, publications: ask me or visit our webpage.

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