Dynamical typicality: What is it and what are its physical and computational implications?

Jochen Gemmer

University of Osnabrueck

Potsdam, Jan. 25th, 2014

Physical implications (Bartsch et al. PRL 102, 110403 (2009))

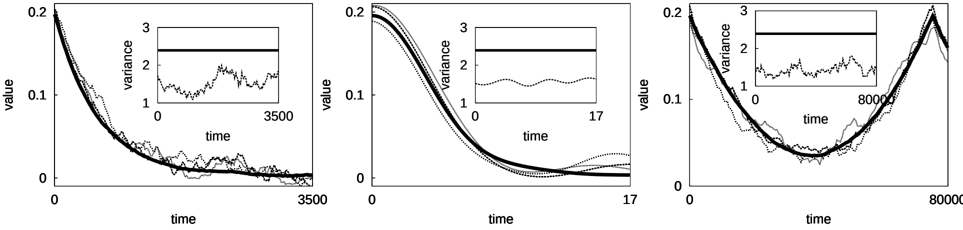
Dynamical typicality in plain language: *A majority of pure quantum states, that feature the same expectation value of some observable at some time, will also do so at later times.*
Dynamical typicality somewhat more precise:

$$\hat{D} := \hat{1} + \epsilon \hat{A}, \qquad \mathrm{Tr}\{\hat{A}\} = 0, \qquad \mathrm{Tr}\{\hat{A}^2\}/d = 1, \qquad \mathrm{Tr}\{\hat{A}^n\}/d = \mathcal{O}(1), n \leq 4$$

set of initial states: $|\omega\rangle = (1/\sqrt{1+\epsilon^2})\hat{D}|\psi\rangle$ $\quad |\psi\rangle$: unit. inv. ensemble

properties of the $|\omega\rangle$-ensemble: $\mathrm{HA}[\langle\omega|\omega\rangle] = 1, \ \mathrm{HV}[\langle\omega|\omega\rangle] = \frac{1}{d+1}\mathcal{O}(1)$

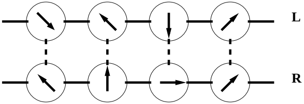
$$\mathrm{HA}[\langle\omega|\hat{A}|\omega\rangle] \approx 2\epsilon, \quad \mathrm{HV}[\langle\omega|\hat{A}(t)|\omega\rangle] \leq \frac{1}{d+1}\mathcal{O}(1)$$



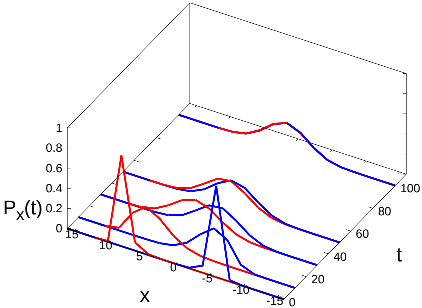
Computational implications (Niemeyer et al., to appear in PRE)

Propagate (Runge-Kutta, etc.) one pure state, get the general dynamics!

weakly coupled, anisotropic Heisenberg chains



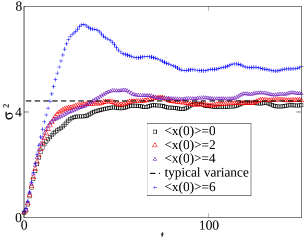
$\hat{x}$: z-magnetization difference between legs

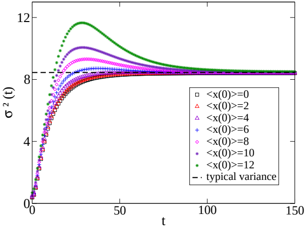


32 spins, $X = 8$, $X = -6$

$$|\omega_X(0)\rangle = e^{-\alpha \hat{H}^2} \hat{P}_x \hat{P}(S_z = 0)|\psi\rangle,$$

variance of x for $N = 16$ (top) and $N = 32$ (bottom)





Size matters !

Propagate some pure states, get information on the Eigenstate Thermalization Hypothesis (ETH)!

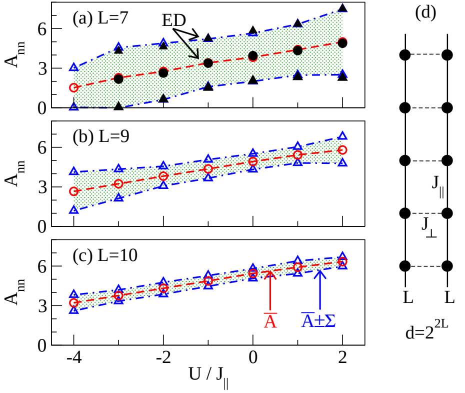
$$\bar{A} \equiv \sum_{n=1}^{d} p_n \langle n|A|n\rangle, \; \Sigma^2 \equiv \sum_{n=1}^{d} p_n \langle n|A|n\rangle^2 - \bar{A}^2$$

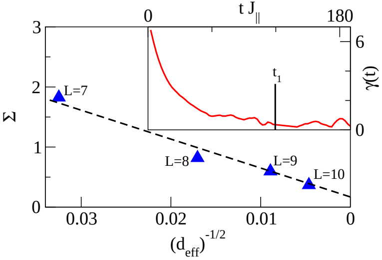
$$P(n) = \frac{1}{Z} e^{\frac{-(E_n - U)^2}{2\sigma^2}} \qquad A : \mathrm{var}(x)$$

$$\gamma(t) := \frac{1}{Z} \mathrm{Tr}\{\hat{A}(t)\hat{A} e^{\frac{-(\hat{H}-U)^2}{2\sigma^2}}\}$$

under "non-resonance condition"

$$\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \gamma(t)\mathrm{d}t \approx \sum_{n=1}^{d} p_n \langle n|A|n\rangle^2$$





ETH applies!

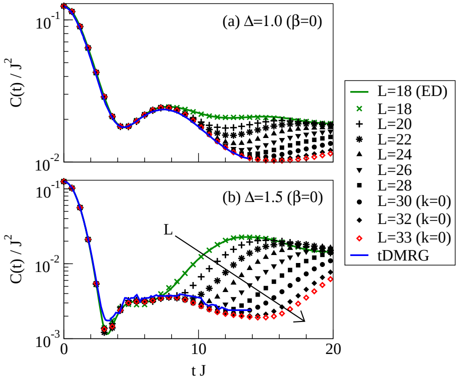
Σ scales approx like $\hat{H}$ was a random matrix.

Computational implications (Steinigeweg et al. arXiv 1312.5319)

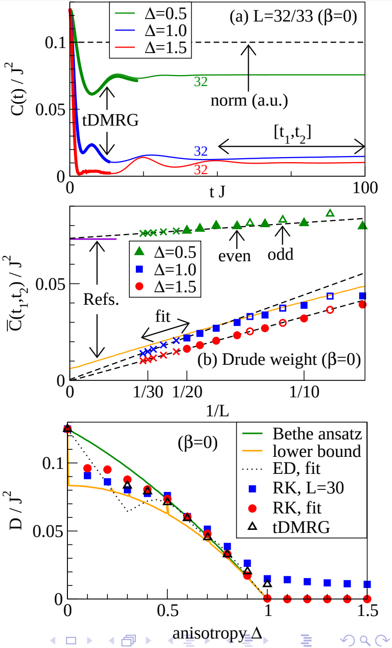
Propagate pure states, get correlation functions!

$$\mathrm{HA}[\langle\psi|\hat{A}(t)|\omega\rangle] = \frac{\epsilon}{d}\mathrm{Tr}\{\hat{A}(t)\hat{A}\}$$

e.g. spin-current in the anisotropic Heisenberg chain $C(t) \propto \mathrm{Tr}\{\hat{J}(t)\hat{J}\}$



Fabian, please correct me !
(when the talk is over)



Thank you for your attention!

The talk itself as well as the mentioned papers may be found on our webpage.