# Emergence of Thermodynamic Behavior in Closed Quantum Systems: A Small Spin System as an Example

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**Thermodynamics**

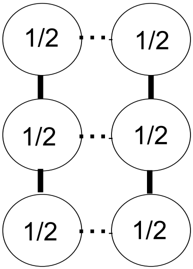
- There are closed, autonomous dynamics with respect to a few macrovariables, e.g. local temperatures.
- Those dynamics feature, up to fluctuations, an attractive fixed point.
- Often dynamics are well described by pertinent stochastic procceses, e.g. of the Ornstein-Uhlenbeck type.

**Quantum Mechanics**

- A priory there are only closed dynamics of extremely many variables, e.g., amplitudes of the wavefunction.
- Schroedinger equation has no attractive fixed point
- There is no direct (widely accepted) stochastic description of unitary quantum dynamics

# Model and observables

**spin-model**




**Heisenberg-type Hamiltonian:** Two weakly coupled, anisotropic Heisenberg chains

$$\hat{H} = \sum_{ij} J_{ij}(\hat{\sigma}_x^i \hat{\sigma}_x^j + \hat{\sigma}_y^i \hat{\sigma}_y^j + 0.6\, \hat{\sigma}_z^i \hat{\sigma}_z^j),$$

where $J_{ij} = 1$ for solid lines, $J_{ij} = 0.15$ for dotted lines and $J_{ij} = 0$ otherwise. Total number of spins $N = 16$. The $z$-component of total magnetization $M_z$ is conserved

We analyze: magnetization difference $\hat{x}$

$$\hat{x} = \sum_{l \in L} \hat{\sigma}_z^l - \sum_{r \in R} \hat{\sigma}_z^r$$

eigenvalues of $\hat{x}$ within the $M_z = 0$ subspace:
$x_i = -\frac{N}{2}, -\frac{N}{2} + 2, \ldots . + \frac{N}{2}$.

We consider more or less random, pure initial states $|\psi\rangle$:

$$|\psi\rangle \propto e^{\epsilon\hat{x}}|\phi\rangle$$

where $|\phi\rangle$ has complex. random, gaussian amplitudes (in any basis).
We compute numerically: mean $m_q(t) := \langle\psi|\hat{x}(t)|\psi\rangle$ and variance $v_q(t) := \langle\psi|\hat{x}^2(t)|\psi\rangle - \langle\psi|\hat{x}(t)|\psi\rangle^2$

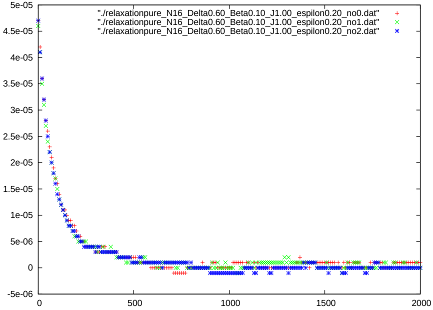
We compare these quantum dynamics to the dynamics as generated by a standard Ornstein-Uhlenbeck stochastic process

$$dX_t = -RX_t dt + DdW_t \quad \text{with} \quad \overline{dW_t dW_s} = \delta_{ts} dt$$

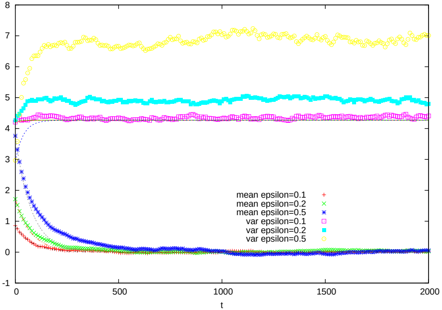
that is, concretely $m_s(t) :=< x(t) >$ and $v_s(t) :=< x^2(t) > - < x(t) >^2$

# Numerical results on the comparison quantum vs. stochastic

$m_q(t)$ **for various** $|\psi\rangle$ **with** $\epsilon = 0.2$




$m_q(t), m_s(t), v_q(t), v_s(t)$ **for various** $\epsilon$



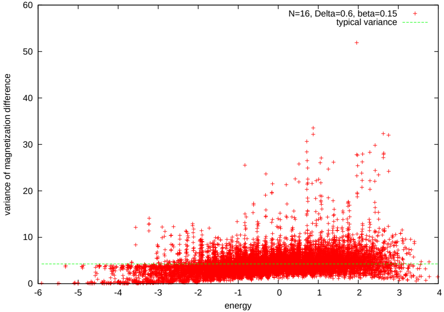

- There seems to be approximate autonomy of the considered variables.
- For small $\epsilon$ the quantum dynamics seem to be in accord with a pertinent Ornstein-Uhlenbeck process.

# Do we understand those numerical findings?

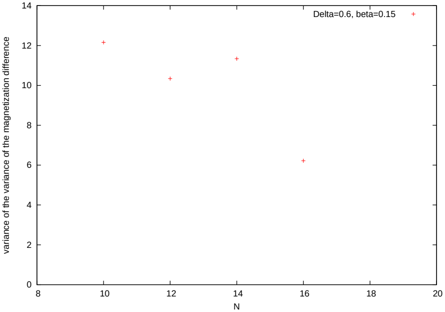
- The relaxation of the observables to more or less universal "equilibrium" is due to the fact that the "eigenstate thermalization hypothesis" (ETH) more or less applies. The ETH claims that single energy eigenstates should exhibit properties similar to those of equilibrium ensembles.
- The similarity of the dynamics of the observables in spite of the initial states being very different is due to "dynamical typicality". This may be expected for any large system.
- The exponential relexation in accord with the Ornstein Uhlenbeck-process is due to the fact that the two halves are weakly coupled and feature relatively broad energy spectra. This makes a Nakajima-Zwanzig type eqaution applicable.

**Thank you for your attention ! (in case I am out of time, otherwise....)**

variances for $N = 16$




skaling of variances

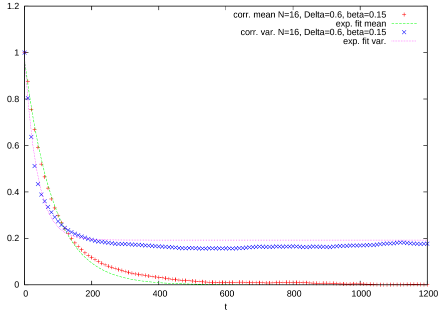



There is a significant "failure" of the ETH up to 16 spins.The ETH applies much less acurately than for states drawn at random (eigenstates of random Hamiltonians). However, one may guess that this failure gradually vanishes for $N \to \infty$

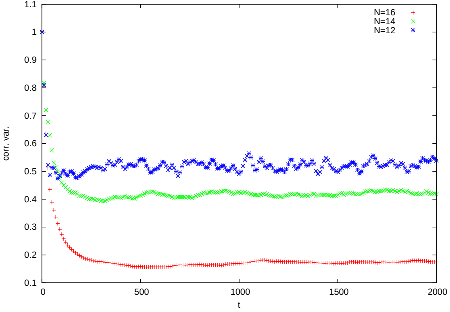
# Concrete quantum dynamics

In order to analyze the typical dynamics for small $\epsilon$ in more detail we compute the temporal correlation functions of $\hat{x}$ and $\hat{x}^2$ i.e., $C_1(t) = \mathrm{Tr}\{\hat{x}(t)\hat{x}\}$ and $C_2(t) = \mathrm{Tr}\{\hat{x}^2(t)\hat{x}^2\}$

**dynamics for $N = 16$**




**variance dynamics for various $N$**




Up to the "stick-effect" there seems to be more or less exponential decay. The ratio of the relaxation rates of second to first moment of the magnetization difference ist found to be 2.2. (For the Ornstein-Uhlenbeck process it is 2.0)