Old and new concepts in transport theory
Scaling behavior of random resistor networks

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Old and new concepts in transport theory

Fouriers Law: A challenge to theorists
Lebowitz et al., *World Scientific* (2000)

“Fourier’s law of heat conduction doesn’t always hold - although precisely when it does, or doesn’t, no one can say.”
M. Buchanan, *naturephysics* **1** (2005)

Two “sorts” of “normal” transport phenomena

“field driven”	“gradient driven”
$\vec{j} = L_F \vec{F}$	$\vec{j} = -L_G \nabla \rho$
electric current induced by an electric field	free diffusion, heat conduction, Brownian motion

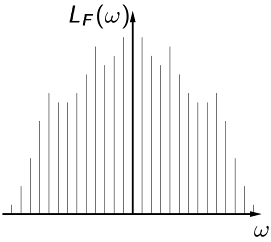
Einstein and Smolouchowski argued in analyzing Brownian motion $L_G \propto L_F T$
But does that hold in general and for all sorts of systems?

Kubo formula: derived for force driven transport

$$\hat{H} = \hat{H}_0 + \hat{U}(t) \quad E = -\nabla U \qquad \frac{\mathrm{d}}{\mathrm{d}t}\hat{U}_H = -E\,\hat{j}_H$$

$$j(\omega) = L(\omega)E(\omega) \quad L(\omega) = \frac{1}{V}\int_0^\infty \mathrm{d}t\, \mathrm{e}^{-\mathrm{i}\omega t} \int_0^\beta \mathrm{d}\tau\, \mathrm{Tr}\{\hat{\rho}_0 \hat{j}(0)\hat{j}(t+\mathrm{i}\tau)\},$$

There is no way to incorporate a density gradient into the Hamiltonian. Nevertheless there are arguments (Luttinger) based on the concept of **"local equilibrium"** claiming that the above formula should also apply to gradient driven transport. Apparently they are not two convincing. (At least not to Lebowitz, Buchanan and me)



finite systems

How can the coefficient for gradient driven transport be extracted for a finite system, i.e., from a discrete spectrum ?

Boltzmann equation

$$\dot{f}(\vec{x}_1, \vec{v}_1, t) + \vec{v}_1 \nabla f(\vec{x}_1, \vec{v}_1, t) =$$
$$\int d\vec{v}_2 d\vec{v}_1' d\vec{v}_2' R(\vec{v}_1, \vec{v}_2 \to \vec{v}_1', \vec{v}_2')(f(\vec{x}_1, \vec{v}_1', t) f(\vec{x}_1, \vec{v}_2', t) - f(\vec{x}_1, \vec{v}_1, t) f(\vec{x}_1, \vec{v}_2, t))$$

Stosszahlansatz (assumption of molecular chaos) breaks time-reversal symmetry, thus the H-theorem applies
Linearization: $f(\vec{x}, \vec{v}, t) = f^0(\vec{v}) + \epsilon f^1(\vec{x}, \vec{v}, t) \quad \Rightarrow$

$$\dot{f}(\vec{x}, \vec{v}, t) + \vec{v} \nabla f(\vec{x}, \vec{v}, t) = \int d\vec{v}' R(\vec{v}, \vec{v}') f(\vec{x}, \vec{v}', t)$$

The linear(ized) Boltzmann equation features diffusive solutions with
$\kappa = \int \vec{v} R^{-1}(\vec{v}, \vec{v}') \vec{v}' f^0(\vec{v}) d\vec{v}' d\vec{v}$
Relaxation time approximation: $R^{-1} = \tau \delta(\vec{v} - \vec{v}') \quad \Rightarrow \kappa = \langle \vec{v}^2 \rangle \tau$

Do quantum systems map onto a classical Boltzmannequation at all, and if so, how?
(There are quantum systems that exhibit diffusive behavior but are not in accord with the Stosszahlansatz)

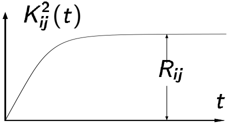
Projection method, reduced autonomous subdynamics

Evolution of observables: $A(t) := \langle \Psi(t) | \hat{A} | \Psi(t) \rangle$

$\Rightarrow$ so called "time convolutionless projection operator method (TCL)" $\Rightarrow$

$$\dot{A}_i = \sum_n \lambda^n \sum_j K_{ij}^n(t) A_j$$

If you strike it lucky $K_{ij}^n(t) \approx 0$ except for $K_{ij}^2(t)$ and $K_{ij}^2(t)$ looks like this:



$$\Rightarrow \quad \dot{A}_i = \sum_j R_{ij} A_j$$

choosing to project onto local occupation numbers P_i one may find:

$$\dot{P}_i = \kappa(P_{i+1} + P_{i+1} - P_i) \quad (\dot{\rho} = \kappa \Delta \rho)$$

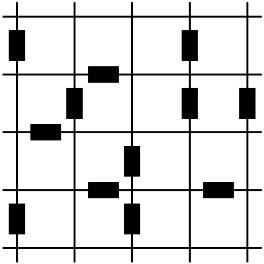
Possilbly the reduced dynamcis result in a diffusion equation

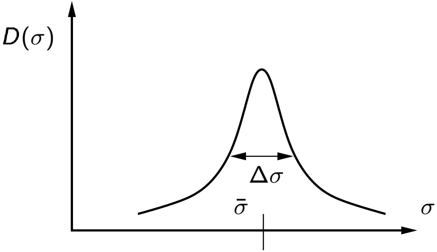
And now for something completely different...

Scaling behavior of random resistor networks

Why do solids typically have welldefined conductivities inspite of impurities, defects, etc?

extremely phenomenological microscopic model of conductivity:

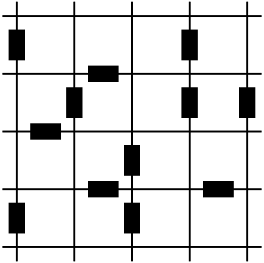




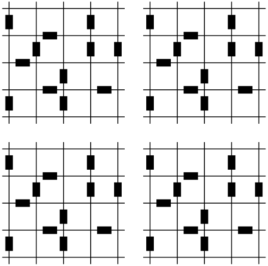
conductivity may depend on individual configuration of defects, etc. but remember:

Law of large numbers!

Law of large numbers and upscaling of networks



$\Rightarrow$

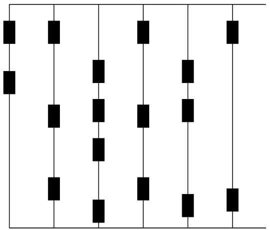


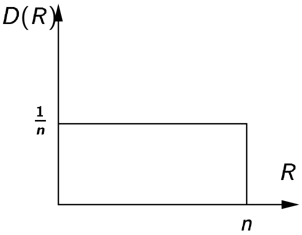
$\bar{\sigma}_n, \ \Delta\sigma_n$ $\qquad$ $\bar{\sigma}_{2n}, \ \Delta\sigma_{2n}$

one expects:

$$\bar{\sigma}_n = \text{const.}, \quad \Delta\sigma_n \longrightarrow 0 \quad \text{for} \quad n \longrightarrow \infty$$

Consider n parallel chains, with a uniform distribution of resistors onto the chains





For such networks neither $\int \sigma D(\sigma) d\sigma$ nor $\int \sigma^2 D(\sigma) d\sigma$ exist. Using the theory of α-stable probability distributions one nevertheless finds:

$$\bar{\sigma}_n \propto \ln(n), \quad \Delta\sigma_n \to \text{const.} \quad \text{for} \quad n \to \infty$$

The law of large numbers does not apply!

Some concluding remarks

- It is unclear (at least to me) what sort of imperfection causes the heat resistance
- It is entirely unclear whether microscopic impurities, defects, etc may be treated like macroscopic resistors.
- There is no result for 2-d networks so far.
- The uniform distribution of resistors onto the chains seems not very plausible. It implies an infinite correlation-length of the resistors along the chains.
- The model can neither explain bi-stability nor hysteresis.
- Possibly it would be more promising to analyze a statistical model featuring some sort of "resistor-ressitor" interaction.