Typicality Approach to Quantum Thermodynamics

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Quantum dynamics:

- $|\psi\rangle$: many parameters
- Schrödinger equation
  $$i\hbar |\dot{\psi}(t)\rangle = \hat{H} |\psi(t)\rangle$$
  features no (attractive) fixpoint
- Von Neumann entropy cannot change: $\dot{S} = 0$
  $$S = -k \text{Tr}\{\hat{\rho} \ln \hat{\rho}\}$$

Thermodynamics:

- few observables $\{A\}$
- Observables: $\{A(t)\} \rightarrow \{A_{eq}\}$
- $\{A_{eq}\}$ are in accord with "maximum entropy principle", “a priori” postulate of equal probabilities, etc. $\Rightarrow \dot{S} \geq 0$

How to overcome these apparent contradictions?
\[ A(\psi) := \langle \psi | \hat{A} | \psi \rangle \text{—"landscape"} \]

“Hilbertspace average” (HA):
\[
\left[ \langle \psi | \hat{A} | \psi \rangle \right] = \frac{\text{Tr}\{\hat{A}\}}{N} \quad (1)
\]

“Hilbertspace variance” (HV):
\[
\left[ \langle \psi | \hat{A} | \psi \rangle^2 \right] - \left[ \langle \psi | \hat{A} | \psi \rangle \right]^2 = \frac{\Delta^2(A)}{(N + 1)} \quad (2)
\]

\( \Delta^2(A) \): “spectral variance” of \( \hat{A} \)

- (2): there exists a super-large bubble for any observable with finite spectrum that is defined on high-dimensional (N) Hilbertspace
- (1): the outcome for \( A(\psi) \) within the super-large bubble is in accord with Boltzmann’s a priori postulate. \( \Rightarrow \) A state \( \psi \) from the super-large bubble cannot be distinguished from equilibrium through observation of \( A(\psi) \).
- the increase of entropy results from an increase of objective quantum uncertainty

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Typicality approach in practical action

Heisenberg spin-ladder $N = 32$

$$\hat{x} = \left( \sum_{l \in L} \hat{\sigma}_z^l - \sum_{r \in R} \hat{\sigma}_z^r \right)$$

$x$: magnetization difference between left and right beam

Data from solving the Schroedinger equation for two pure initial states!