Typicality Approach to Quantum Thermodynamics

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Quantum dynamics:

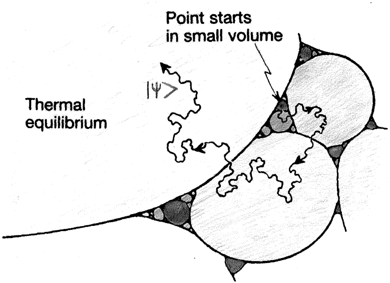
- $|\psi\rangle$: many parameters
- Schrödinger equation $i\hbar|\dot{\psi}(t)\rangle = \hat{H}|\psi(t)\rangle$ features no (attractive) fixpoint
- Von Neumann entropy cannot change: $\dot{S} = 0$, $S = -k\mathrm{Tr}\{\hat{\rho}\ln\hat{\rho}\}$

Thermodynamics:

- few observables $\{A\}$
- Observables: $\{A(t)\} \rightarrow \{A_{eq}\}$
- $\{A_{eq}\}$ are in accord with "maximum entropy principle", "a priori" postulate of equal probabilities, etc. $\Rightarrow \dot{S} \geq 0$

How to overvcome these apparent contradictions?

$A(\psi) := \langle\psi|\hat{A}|\psi\rangle$-**"landscape"**



"Hilbertspace average" (HA):

$$[\![\langle\psi|\hat{A}|\psi\rangle]\!] = \frac{\mathrm{Tr}\{\hat{A}\}}{N} \qquad (1)$$

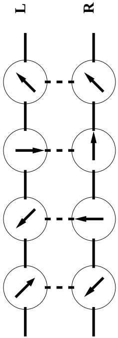
"Hilbertspace variance" (HV):

$$[\![\langle\psi|\hat{A}|\psi\rangle^2]\!] - [\![\langle\psi|\hat{A}|\psi\rangle]\!]^2 = \frac{\Delta^2(A)}{(N+1)} \qquad (2)$$

$\Delta^2(A)$: "spectral variance" of $\hat{A}$

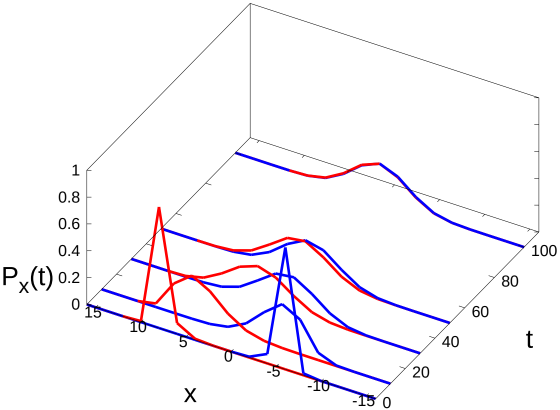
- (2): there exists a super-large bubble for any observable with finite spectrum that is defined on high-dimensional (N) Hilbertspace
- (1): the outcome for $A(\psi)$ wihtin the super-large bubble is in accord with Boltzmann's a priori postulate. $\Rightarrow$ A state ψ from the super-large bubble cannot be distinguished from equilibrium through observation of $A(\psi)$.
- the increase of entropy results from an increase of objective quantum uncertainty

Heisenberg
spin-ladder $N = 32$



$$\hat{x} = \left(\sum_{l \in L} \hat{\sigma}_z^l - \sum_{r \in R} \hat{\sigma}_z^r \right)$$

x: magnetization difference between left and right beam



Data from solving the Schroedinger equation for two pure initial states!