Relevance of the Eigenstate Thermalization Hypothesis for Initial State Independent Equilibration

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Facts, Notions, Concepts (and cups of coffee....)

Temperature differences between macroscopic objects in energy exchanging contact are expected to vanish, irrespective of their initial values.

- Eigenstate Thermalization Hypothesis (ETH): "cloud width" $\Sigma(\hat{D}, \hat{H})$ small

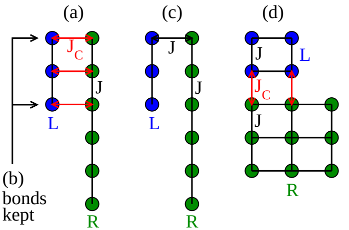
$$\Sigma^2 \equiv \sum_{n=1}^{d} p_n \langle n|\hat{D}|n\rangle^2 - \bar{D}^2 \quad \bar{D} \equiv \sum_{n=1}^{d} p_n \langle n|\hat{D}|n\rangle \quad \hat{H}|n\rangle = E_n$$

  p_n probability distribution, sharply peaked at some $E_n = \bar{E}$
- Initial state independence (ISI): Expectation values of some observable $\hat{D}$ relax towards a common value irrespective of their initial values.
- non-resonance condition (NRC): any difference between two eigenvalues of $\hat{H}$ occurs only once.
- Given the NRC holds and the ETH applies $\Rightarrow$ ISI follows for all possible initial states with sufficiently broad energy distributions
- If the ETH does not apply there may or may not be ISI, depending on the initial state.

Is the ETH (in the above sense) physically imperative for ISI of energy differences?

Maybe the ETH always applies to “Temperature” Relaxation?

model:
weakly coupled, anisotropic Heisenberg chains, $N_R = 2N_L$



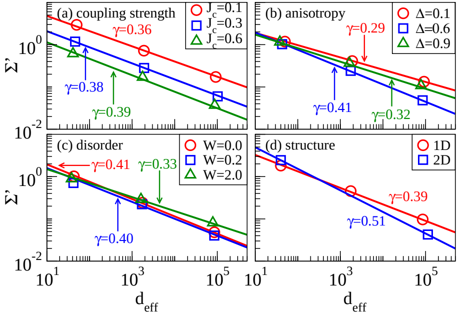
pieces of the Hamiltonian:

$$\hat{S}_x^\alpha \hat{S}_x^\beta + \hat{S}_y^\alpha \hat{S}_y^\beta + \Delta\, \hat{S}_z^\alpha \hat{S}_z^\beta + B_\alpha \hat{S}_z^\alpha$$

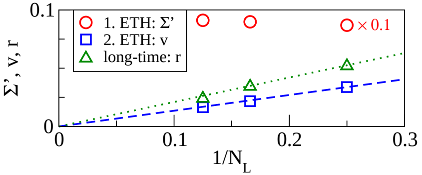
observable: energy difference:

$$\hat{D} = \hat{H}_L - \hat{H}_R$$

scaling of $\Sigma \propto d_{eff}^{-\gamma}$ for $N_L = 4, 6, 8(9)$



What about clean Heisenberg chain?



this indicates $\Sigma = const$

Do energy differences in the clean Heisenberg chain not relax ISI?

What initial state? $\Rightarrow$ microcanonical observable displaced state (MOD) (no "quench")

$$\hat{\rho}(0) = \rho_{\mathrm{MOD}}(\chi, \sigma, d') :\propto e^{-([\hat{H}-0]^2+\chi^2[\hat{D}-d']^2)/2\sigma^2} .$$

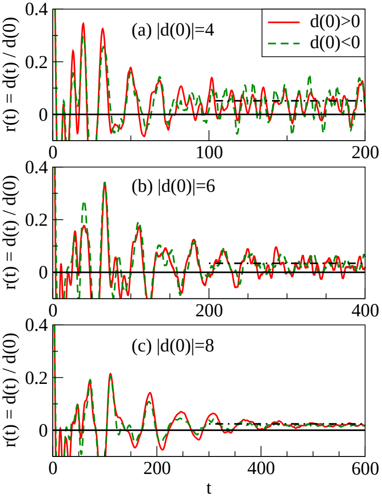
choosing χ, σ, d' carefully we are able to prepare states with $\Delta E \approx 0.3$ and $d(0) = \pm N_L$ with $d(t) := \langle \hat{D}(t) \rangle$ (overall energy scale ca. $3N_L$)

"stick effect": it looks like $d(t) \to r(N_L) d(0)$ where $r(N_L)$ is a constant which is independent of $d(0)$
Does the stick effect vanish with increasing size?

Besides, Σ is not a dimensionless quantity, so what is "small"?
Define, just for fun: $v^2 = \Sigma^2/\delta^2$ with

$$\delta^2 = \langle \hat{D}^2 \rangle_0 - \langle \hat{D} \rangle_0^2$$

with $\langle \cdots \rangle_0 = \mathrm{Tr}\{\cdots \hat{\rho}_{\mathrm{MOD}}(\chi = 0, \sigma, d')\}$



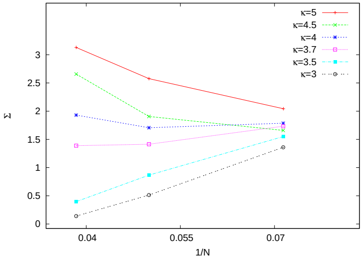
...seems they do !

....and v rather than Σ detects that.

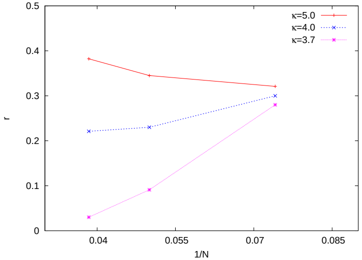
What about strong couplings?

Same model (a), strong interchain couplings $J_c = (3...5)J$

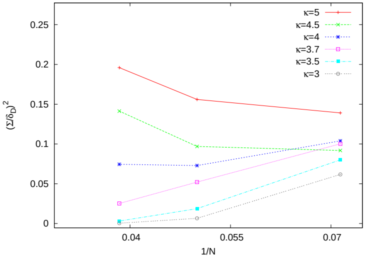
Σ may increase



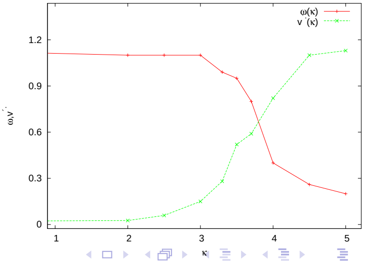
serious stick effect, "detected" by v



even v may increase



integrable again ?



Thank you for your attention!

The talk itself as well as related papers from our group may be found on our webpage.