Impact of the eigenstate thermalization hypothesis on the relaxation of significantly off-equilibrium initial states

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Temperature differences between macroscopic objects in energy exchanging contact are expected to vanish, irrespective of their initial values.

Applicability of the eigenstate thermalization (ETH), is sufficient (but not necessary) for initial state independent (ISI) relaxation of observable $\hat{D}$

$$\text{ETH : } \Sigma(\hat{D}, \hat{H}) \text{ small !} \quad \Sigma^2 \equiv \sum_{n=1}^{d} p_n \langle n | \hat{D} | n \rangle^2 - \bar{D}^2 \quad \bar{D} \equiv \sum_{n=1}^{d} p_n \langle n | \hat{D} | n \rangle$$

$p_n$ probability distribution, sharply peaked at some $E_n = \bar{E}$

pieces of the Hamiltonian:

$$\hat{S}_x^\alpha \hat{S}_x^\beta + \hat{S}_y^\alpha \hat{S}_y^\beta + \Delta \hat{S}_z^\alpha \hat{S}_z^\beta + B_{\alpha} \hat{S}_z$$

observable: energy difference:

$$\hat{D} = \hat{H}_L - \hat{H}_R$$

scaling of $\Sigma \propto d_{\text{eff}}^{-\gamma}$ for $N = 12, 28, 24, (25)$

(a) coupling strength
(b) anisotropy
(c) disorder
(d) structure

- $\gamma = 0.36$
- $\gamma = 0.38$
- $\gamma = 0.39$
- $\gamma = 0.40$
- $\gamma = 0.33$
- $\gamma = 0.41$
- $\gamma = 0.32$
- $\gamma = 0.41$
- $\gamma = 0.39$
Do energy differences in the clean Heisenberg chain not relax ISI?

What initial state? ⇒ microcanonical observable displaced state (MOD) (no “quench”)

\[ \hat{\rho}(0) = \rho_{\text{MOD}} : \propto e^{-\frac{(\hat{H} - E')^2}{2\sigma^2} - \frac{(\hat{D} - d')^2}{2\chi^2}}. \]

This way we are able to “prepare” states that have the biggest part of the energy in the left/right part while, still being energetically restricted to ca. 1/100 of the full spectrum (“energy shell”)

**dynamics:** \( d(t) := \langle \hat{D}(t) \rangle \) “stick effect”: it looks like \( d(t) \to r(N_L)d(0) \) where \( r(N_L) \) is a constant which is independent of \( d(0) \)

Does the stick effect vanish with increasing size?

Besides, \( \Sigma \) is not a dimensionless quantity, so what is “small”?

Define, just for fun: \( \nu^2 = \frac{\Sigma^2}{\delta^2} \) with

\[ \delta^2 = \langle \hat{D}^2 \rangle_{\text{shell}} - \langle \hat{D} \rangle_{\text{shell}}^2 \]
“More integrable” than clean Heisenberg chain?

Same model (a), strong interchain couplings $J_c = (3...5)J$

$\Sigma$ may increase

serious stick effect, “detected” by $v$

even $v$ may increase

level statistics and stick effect
Thank you for your attention!

The talk itself as well as related papers from our group may be found on our webpage.