Typicality Approach to Transport in 1-D Quantum magnets

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and Jochen Gemmer**

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What is it all about?

You want: to calculate current autocorrelation functions and their integrals in order do obtain transport coefficents from linear response theory

$$\sigma \propto \int_0^\infty \langle \hat{j}(t)\hat{j} \rangle dt,$$

preferably for very large sytems to get rid of finite size effects.

You cannot: propagate operators like $\hat{j}$ or calculate thermal states $\propto e^{-\beta \hat{H}}$ for large enough systems due to limited computing power/memory.

Solution : Only propagate one (a few) pure states!

How can that work?

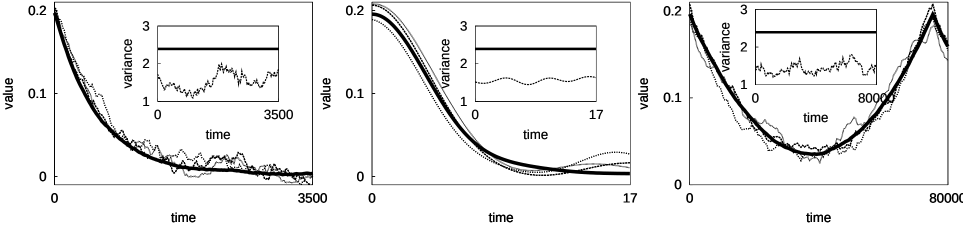
How can (non-equilibrium) thermodynamics get away with taking so few observables into account?

One possible answer: quantum details very often don't matter, states behave in a "typical" way.

Typicality in plain language: *Very many individual pure quantum states may "mimic" the dynamics of an ensemble to a very good degree*

$$\text{typicality in formulas} \quad \mathrm{HV}[\langle\omega|\hat{A}(t)|\omega\rangle] \leq \frac{1}{d+1}\mathcal{O}(1)$$

Dynamics of expectation values $\langle\omega|\hat{A}(t)|\omega\rangle$ for pure states $|\omega\rangle$ that feature all the same $\langle\omega|\hat{A}(0)|\omega\rangle$ but are "random" otherwise:



Bartsch et al.: PRL 102, 110403 (2009)

The recipe

How to get an approximation to $\text{Tr}\{(\hat{j}(t)\hat{j} + \hat{j}\hat{j}(t))e^{-\beta\hat{H}}\}$:

Take a state $|\psi\rangle$ with real and imaginary parts of all amplitudes (w.r.t. a basis of your choice) drawn as independent random Gaussian numbers, normalize.

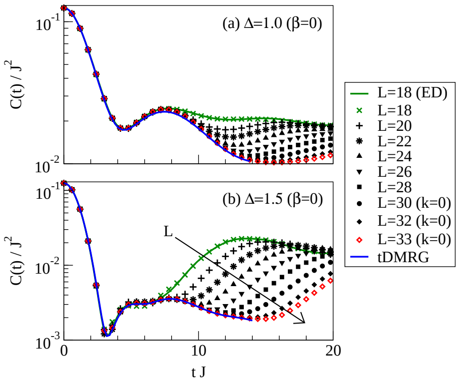
Propagate in imaginary time, i.e., compute $e^{-\beta\hat{H}}|\psi\rangle$, normalize. Call result $|\psi'\rangle$, Use Runge-Kutta, Chebeyshev, etc.

Propagate in real time and apply $\hat{j}$ in two ways: $|\mu\rangle := \hat{j}e^{-i\hat{H}t}|\psi'\rangle$ and $|\nu\rangle := e^{-i\hat{H}t}\hat{j}|\psi'\rangle$.

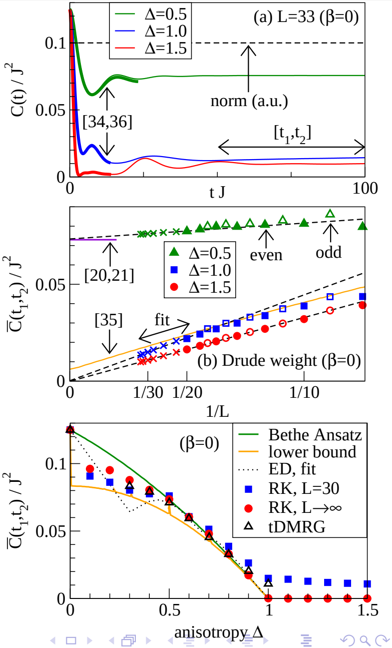
Compute $\text{Re}(\langle\mu(t)|\nu(t)\rangle)$.....You are done!

Spin Transport in the clean Heisenberg chain $\beta = 0$

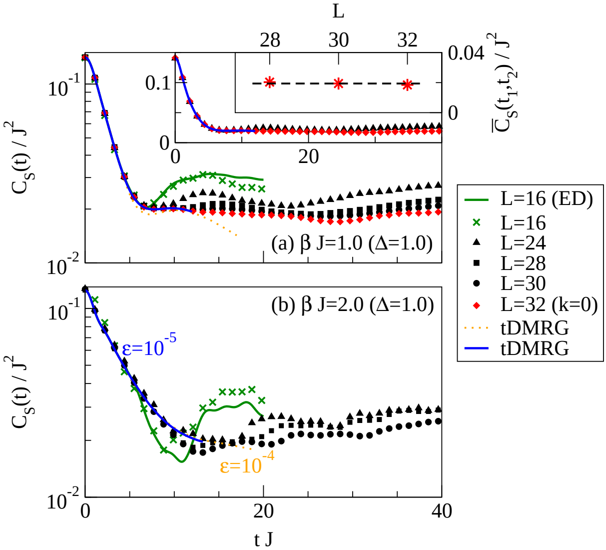
ballistic or diffusive ?



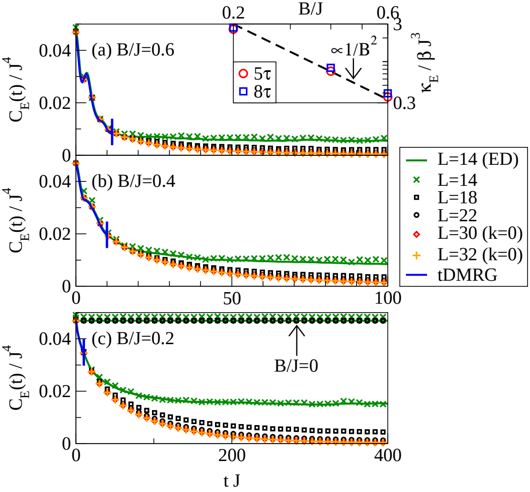
Steinigeweg et al.:
PRB 91, 104404 (2015)
PRL 112 (12), 120601 (2014)



Spin Transport in the clean Heisenberg chain $\beta \neq 0$



most likely ballistic at $\Delta = 1$

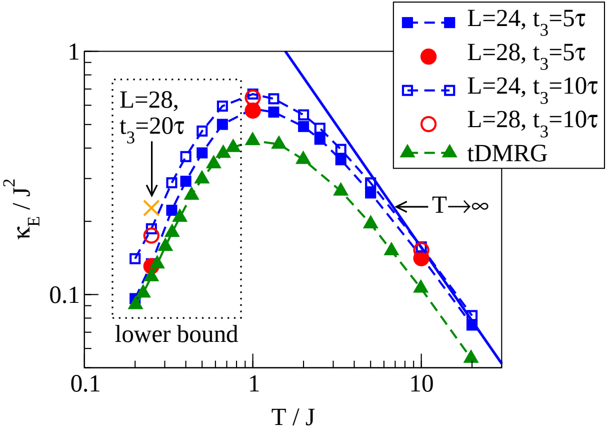


staggered field breaks integrabilty, so no Drude weight is expected, rather we want transport coefficients.

staggerd field quantified by B, anisotropy $\Delta = 0.5$

similar calculations for varying Δ eventually suggest:

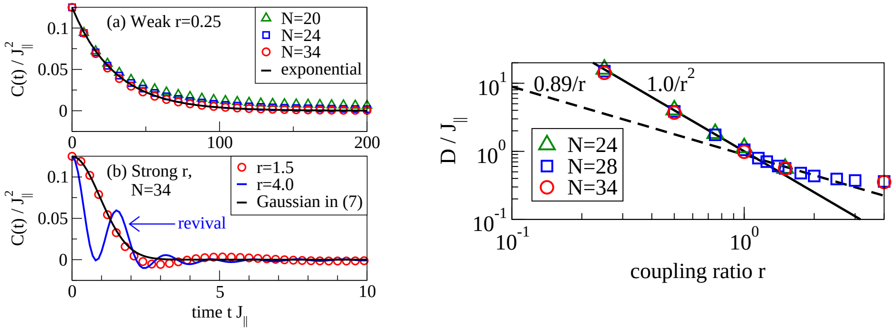
$$\kappa_E \propto \frac{(1 + 2\Delta^2)^2}{(B\Delta)^2}$$



Obtaining quantitative transport coefficients for low temperatures is hard !

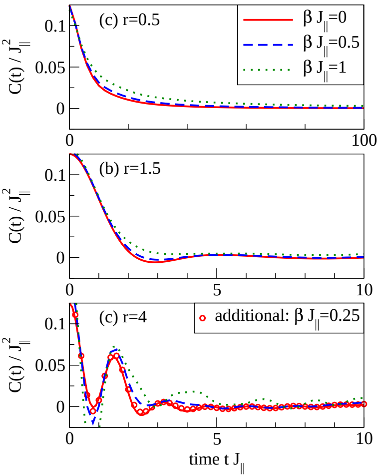
Spin Transport in the XX-ladder $\beta = 0$

r: ratio of rung-coupling to leg-coupling. $r = 0$: mapable to free fermions



For small r the dynamics appears to be nicely in accord with a simple Drude model.

Steinigewg, Heidrich-Meisner et. al.: PRB 90, 094417 (2014)



Maybe obtaining quantitative transport coefficients for low temperatures is not all that hard ?

Thank you for your attention!

The talk itself as well as the mentioned papers may soon be found on our webpage.

(If you are in a hurry: Check Robin Steinigewegs webpage)